

Divergent Box Integral 11: $I_4^{\{D=4-2\epsilon\}}(0, m_3^2, p_3^2, m_4^2; s_{12}, s_{23}; 0, 0, m_3^2, m_4^2)$

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The result for this integral (see **figure**) in the unphysical region $s_{12}, s_{23} < 0, p_3^2 \leq 0$ is[?],

$$\begin{aligned} I_4^{\{D=4-2\epsilon\}}(0, m_3^2, p_3^2, m_4^2; s_{12}, s_{23}; 0, 0, m_3^2, m_4^2) &= \frac{1}{(m_3^2 - s_{12})(m_4^2 - s_{23})} \\ &\times \left[\frac{1}{\epsilon^2} - \frac{1}{\epsilon} \ln \left(\frac{(m_4^2 - s_{23})(m_3^2 - s_{12})}{m_3 m_4 \mu^2} \right) + 2 \ln \left(\frac{m_3^2 - s_{12}}{m_3 \mu} \right) \ln \left(\frac{m_4^2 - s_{23}}{m_4 \mu} \right) \right. \\ &\left. - \frac{\pi^2}{2} + \ln^2 \left(\frac{m_3}{m_4} \right) - \frac{1}{2} \ln^2 \left(\frac{\gamma_{34}^+}{\gamma_{34}^+ - 1} \right) - \frac{1}{2} \ln^2 \left(\frac{\gamma_{34}^-}{\gamma_{34}^- - 1} \right) \right] + \mathcal{O}(\epsilon) \end{aligned}$$

where

$$\gamma_{ij}^\pm = \frac{1}{2} \left[1 - \frac{m_i^2 - m_j^2}{p_3^2} \pm \sqrt{\left(1 - \frac{m_i^2 - m_j^2}{p_3^2}\right)^2 - \frac{4m_j^2}{p_3^2}} \right]$$

and $\gamma_{ij}^+ + \gamma_{ji}^- = 1$.

In the limit $p_3^2 \rightarrow 0$

$$\frac{\gamma_{34}^+}{\gamma_{34}^+ - 1} \rightarrow 1 + \mathcal{O}(p_3^2), \quad \frac{\gamma_{34}^-}{\gamma_{34}^- - 1} \rightarrow \frac{m_4^2}{m_3^2} + \mathcal{O}(p_3^2)$$

and this expression reduces to the form given in Eq. (6.77) of Höpker [?].

$$\begin{aligned} I_4^{\{D=4-2\epsilon\}}(0, m_3^2, 0, m_4^2; s_{12}, s_{23}; 0, 0, m_3^2, m_4^2) &= \frac{1}{(m_3^2 - s_{12})(m_4^2 - s_{23})} \\ &\times \left[\frac{1}{\epsilon^2} - \frac{1}{\epsilon} \ln \left(\frac{(m_4^2 - s_{23})(m_3^2 - s_{12})}{m_3 m_4 \mu^2} \right) \right. \\ &\left. + 2 \ln \left(\frac{m_3^2 - s_{12}}{m_3 \mu} \right) \ln \left(\frac{m_4^2 - s_{23}}{m_4 \mu} \right) - \frac{\pi^2}{2} - \ln^2 \left(\frac{m_3}{m_4} \right) \right] + \mathcal{O}(\epsilon) \end{aligned}$$

A limit of this integral $I_4^{\{D=4-2\epsilon\}}(0, m^2, 0, m^2; s_{12}, s_{23}; 0, 0, m^2, m^2)$ is given in Eq. (6.70) of ref. [?].

A limit of this integral $I_4^{\{D=4-2\epsilon\}}(0, m_3^2, 0, m_4^2; s_{12}, s_{23}; 0, 0, m_3^2, m_4^2)$ is given in Eq. (6.77) of ref. [?].

A limit of this integral $I_4^{\{D=4-2\epsilon\}}(0, m^2, p_3^2, m^2; s_{12}, s_{23}, 0, 0, m^2, m^2)$ can be obtained from Eq.(A4), Equation 3 of ref. [?]

References

- [1] R. K. Ellis and G. Zanderighi, “Scalar one-loop integrals for QCD,” [arXiv:0712.1851 \[hep-ph\]](#)
- [2] R. Höpker, Hadroproduction and decay of squarks and gluinos, (in german), DESY Internal report DESY-T-96-02, ([Relevant excerpt](#))
- [3] W. Beenakker, S. Dittmaier, M. Kramer, B. Plumper, M. Spira and P. M. Zerwas, Nucl. Phys. B **653**, 151 (2003) [[arXiv:hep-ph/0211352](#)]