**Divergent Box Integral 13:** $I_4^{\{D=4-2\epsilon\}}(0,p_2^2,p_3^2,p_4^2;s_{12},s_{23};0,0,m_3^2,m_4^2)$

Page contributed by R.K. Ellis

The result for this integral is[**?**], (see figure) See the file on notation.

$$
\begin{aligned}
I_4^{\{D=4-2\epsilon\}}(0,p_2^2,p_3^2,p_4^2;s_{12},s_{23};0,0,m_3^2,m_4^2) &= \frac{1}{\Delta}\Bigg[\frac{1}{\epsilon}\ln\left(\frac{(m_3^2-p_2^2)(m_4^2-p_4^2)}{(m_3^2-s_{12})(m_4^2-s_{23})}\right) \\
- & 2\,\mathrm{Li}_2\left(1-\frac{(m_3^2-p_2^2)}{(m_3^2-s_{12})}\right)-\mathrm{Li}_2\left(1-\frac{(m_3^2-p_2^2)}{(m_4^2-s_{23})}\frac{\gamma_{34}^+}{\gamma_{34}^+-1}\right)-\mathrm{Li}_2\left(1-\frac{(m_3^2-p_2^2)}{(m_4^2-s_{23})}\frac{\gamma_{34}^-}{\gamma_{34}^--1}\right) \\
- & 2\,\mathrm{Li}_2\left(1-\frac{(m_4^2-p_4^2)}{(m_4^2-s_{23})}\right)-\mathrm{Li}_2\left(1-\frac{(m_4^2-p_4^2)}{(m_3^2-s_{12})}\frac{\gamma_{43}^+}{\gamma_{43}^+-1}\right)-\mathrm{Li}_2\left(1-\frac{(m_4^2-p_4^2)}{(m_3^2-s_{12})}\frac{\gamma_{43}^-}{\gamma_{43}^--1}\right) \\
+ & 2\,\mathrm{Li}_2\left(1-\frac{(m_3^2-p_2^2)(m_4^2-p_4^2)}{(m_3^2-s_{12})(m_4^2-s_{23})}\right)+2\ln\left(\frac{m_3^2-s_{12}}{\mu^2}\right)\ln\left(\frac{m_4^2-s_{23}}{\mu^2}\right) \\
- & \ln^2\left(\frac{m_3^2-p_2^2}{\mu^2}\right)-\ln^2\left(\frac{m_4^2-p_4^2}{\mu^2}\right)+\ln\left(\frac{m_3^2-p_2^2}{m_4^2-s_{23}}\right)\ln\left(\frac{m_3^2}{\mu^2}\right)+\ln\left(\frac{m_4^2-p_4^2}{m_3^2-s_{12}}\right)\ln\left(\frac{m_4^2}{\mu^2}\right) \\
- & \frac{1}{2}\ln^2\left(\frac{\gamma_{34}^+}{\gamma_{34}^+-1}\right)-\frac{1}{2}\ln^2\left(\frac{\gamma_{34}^-}{\gamma_{34}^--1}\right)\Bigg]+\mathcal{O}(\epsilon)
\end{aligned}
$$

where

$$
\begin{aligned}
\Delta &= (s_{12}s_{23}-m_3^2s_{23}-m_4^2s_{12}-p_2^2p_4^2+m_3^2p_4^2+m_4^2p_2^2) \\
&= (m_3^2-s_{12})(m_4^2-s_{23})-(m_3^2-p_2^2)(m_4^2-p_4^2)
\end{aligned}
$$

In the limit $p_3^2 \to 0$ this simplifies slightly to

$$
\begin{aligned}
I_4^{\{D=4-2\epsilon\}}(0,p_2^2,0,p_4^2;s_{12},s_{23};0,0,m_3^2,m_4^2) &= \frac{1}{\Delta}\Bigg[\frac{1}{\epsilon}\ln\left(\frac{(m_3^2-p_2^2)(m_4^2-p_4^2)}{(m_3^2-s_{12})(m_4^2-s_{23})}\right) \\
- & 2\,\mathrm{Li}_2\Big(1-\frac{(m_3^2-p_2^2)}{(m_3^2-s_{12})}\Big) - \mathrm{Li}_2\Big(1-\frac{(m_3^2-p_2^2)}{(m_4^2-s_{23})}\Big) - \mathrm{Li}_2\Big(1-\frac{m_4^2}{m_3^2}\frac{(m_3^2-p_2^2)}{(m_4^2-s_{23})}\Big) \\
- & 2\,\mathrm{Li}_2\Big(1-\frac{(m_4^2-p_4^2)}{(m_4^2-s_{23})}\Big) - \mathrm{Li}_2\Big(1-\frac{(m_4^2-p_4^2)}{(m_3^2-s_{12})}\Big) - \mathrm{Li}_2\Big(1-\frac{m_3^2}{m_4^2}\frac{(m_4^2-p_4^2)}{(m_3^2-s_{12})}\Big) \\
+ & 2\,\mathrm{Li}_2\Big(1-\frac{(m_3^2-p_2^2)(m_4^2-p_4^2)}{(m_3^2-s_{12})(m_4^2-s_{23})}\Big) + 2\ln\left(\frac{m_3^2-s_{12}}{\mu^2}\right)\ln\left(\frac{m_4^2-s_{23}}{\mu^2}\right) \\
- & \ln^2\left(\frac{m_3^2-p_2^2}{\mu^2}\right) - \ln^2\left(\frac{m_4^2-p_4^2}{\mu^2}\right) + \ln\left(\frac{m_3^2-p_2^2}{m_4^2-s_{23}}\right)\ln\left(\frac{m_3^2}{\mu^2}\right) + \ln\left(\frac{m_4^2-p_4^2}{m_3^2-s_{12}}\right)\ln\left(\frac{m_4^2}{\mu^2}\right) \\
- & \frac{1}{2}\ln^2\left(\frac{m_4^2}{m_3^2}\right)\Bigg] + \mathcal{O}(\epsilon)
\end{aligned}
$$

A limit of this integral, $I_4^{\{D=4-2\epsilon\}}(0,p^2,0,p^2;s_{12},s_{23};0,0,m^2,m^2)$ is given in Eq. (6.72) of ref. [**?**]

A limit of this integral, $I_4^{\{D=4-2\epsilon\}}(0,m_4^2,0,m_3^2;s_{12},s_{23};0,0,m_3^2,m_4^2)$ is given in Eq. (6.79) of ref. [**?**]

## References


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