Divergent Box Integral 15: $I_4^{\{D=4-2\epsilon\}}(m_2^2, p_2^2, p_3^2, m_4^2, p_4^2; s_{12}, s_{23}; 0, m_2^2, 0, m_4^2)$

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We can calculate this IR divergent box integral from Eq. (2.11) of ref.[1], using the simple replacement rule $\ln\lambda^2 \longrightarrow \frac{r_\Gamma}{\epsilon} + \ln\mu^2$. We obtain

$$
\begin{aligned}
& I_4^{\{D=4-2\epsilon\}}(m_2^2, p_2^2, p_3^2, m_4^2; t, s; 0, m_2^2, 0, m_4^2) \\
= & \frac{x_s}{m_2 m_4 t(1-x_s^2)} \Bigg\{ \ln x_s \left[-\frac{1}{\epsilon} - \frac{1}{2}\ln x_s - \ln\left(\frac{\mu^2}{m_2 m_4}\right) - \ln\left(\frac{m_2^2 - p_2^2}{-t}\right) - \ln\left(\frac{m_4^2 - p_3^2}{-t}\right) \right] \\
- & \mathrm{Li}_2(1 - x_s^2) + \frac{1}{2}\ln^2 y + \sum_{\rho=\pm 1} \mathrm{Li}_2(1 - x_s y^\rho) \Bigg\}
\end{aligned}
$$

Note the reversal of the arguments s, t to conform with the notation of [1].

$$
y = \frac{m_2}{m_4} \frac{(m_4^2 - p_3^2)}{(m_2^2 - p_2^2)}
$$

The variable x_s is defined in terms of the function K, such that $x_s = -K(s + i\varepsilon, m_2, m_4)$ and K is given by

$$
\begin{aligned}
K(z, m, m') &= \frac{1 - \sqrt{1 - 4mm'/[z - (m - m')^2]}}{1 + \sqrt{1 - 4mm'/[z - (m - m')^2]}} \qquad z \neq (m - m')^2 \\
K(z, m, m') &= -1 \qquad z = (m - m')^2
\end{aligned}
$$

In the limit $x_s \longrightarrow 1$ (i.e $s = (m_2 - m_4)^2$) we obtain

$$
\begin{aligned}
& I_4^{\{D=4-2\epsilon\}}(p_1^2, m_2^2, m_4^2, p_4^2; t, s; 0, m_2^2, 0, m_4^2) = \frac{1}{2 m_2 m_4 t} \\
\times & \left\{ \frac{1}{\epsilon} + \ln\left(\frac{\mu^2}{m_2 m_4}\right) + \ln\left(\frac{m_2^2 - p_2^2}{-t}\right) + \ln\left(\frac{m_4^2 - p_3^2}{-t}\right) - 2 - \frac{1 + y}{(1 - y)} \ln y \right\}
\end{aligned}
$$

In the limit $p_2^2 \to m_2^2$ we obtain

$$I_4^{\{D=4-2\epsilon\}}(m_2^2, m_2^2, p_3^2, m_4^2; t, s; 0, m_2^2, 0, m_4^2) = \frac{x_s}{m_2 m_4 t(1-x_s^2)}$$
$$\times \left\{ \ln x_s \left[-\frac{1}{\epsilon} - \ln x_s - \ln\left(\frac{\mu^2}{m_4^2}\right) - 2\ln\left(\frac{m_4^2 - p_3^2}{-t}\right) \right] - \mathrm{Li}_2(1-x_s^2) \right\}$$

References

[1] W. Beenakker and A. Denner, Nucl. Phys. B **338**, 349 (1990). Inspire