Divergent Box Integral 2: $I_4^{D=4-2\epsilon}(0,0,0,p_4^2;s_{12},s_{23};0,0,0,0)$

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The result for the box integral (see figure) with one off-shell leg in the unphysical region ($s_{12}<0, s_{23}<0, p_4^2<0$) is [1, 2]

$$I_4^{D=4-2\epsilon}(0,0,0,p_4^2;s_{12},s_{23};0,0,0,0) = \frac{\mu^{2\epsilon}}{s_{12}s_{23}}$$
$$\times \quad \Big[\frac{2}{\epsilon^2}\Big((-s_{12})^{-\epsilon}+(-s_{23})^{-\epsilon}-(-p_4^2)^{-\epsilon}\Big) - 2\,\mathrm{Li}_2(1-\frac{p_4^2}{s_{12}}) - 2\,\mathrm{Li}_2(1-\frac{p_4^2}{s_{23}})$$
$$- \quad \ln^2\left(\frac{-s_{12}}{-s_{23}}\right) - \frac{\pi^2}{3}\Big] + \mathcal{O}(\epsilon)\,.$$

By inverting the dilogarithms we can write this as

$$I_4^{D=4-2\epsilon}(0,0,0,p_4^2;s_{12},s_{23};0,0,0,0) = \frac{\mu^{2\epsilon}}{s_{12}s_{23}}\Big[\frac{2}{\epsilon^2}\Big((-s_{12})^{-\epsilon}+(-s_{23})^{-\epsilon}-(-p_4^2)^{-\epsilon}\Big) + 2\,\mathrm{Ls}_{-1}(\frac{s_{12}}{p_4^2},\frac{s_{23}}{p_4^2})\Big] + \mathcal{O}(\epsilon)\,.$$

The function Ls_{-1} is defined as

$$\mathrm{Ls}_{-1}(r_1,r_2) = \mathrm{Li}_2(1-r_1) + \mathrm{Li}_2(1-r_2) + \ln(r_1)\ln(r_2) - \frac{\pi^2}{6}$$

See the file on notation. The correct analytic continuation to the physical region can be obtained from this expression by substituting $s_{ij}\to s_{ij}+i\varepsilon, p_4^2\to p_4^2+i\varepsilon,$.

An alternative formulation[3] makes the continuation quite explicit

$$f^{1m} = \frac{s_{12}+s_{23}-p_4^2}{s_{12}s_{23}}$$

The one−mass box scalar integral is

$$I_4^{D=4-2\epsilon}(0,0,0,p_4^2;s_{12},s_{23};0,0,0,0) = \frac{1}{s_{12}\,s_{23}}\Big[\frac{2}{\epsilon^2}\Big((-s_{12}-i\varepsilon)^{-\epsilon}+(-s_{23}-i\varepsilon)^{-\epsilon}-(-p_4^2-i\varepsilon)^{-\epsilon}\Big) +2\,\mathrm{Li}_2\Big(1-(s_{12}+\mathrm{i}\varepsilon)\,f^{1m}\Big)+2\,\mathrm{Li}_2\Big(1-(s_{23}+\mathrm{i}\varepsilon)\,f^{1m}\Big) -2\,\mathrm{Li}_2\Big(1-(p_4^2+\mathrm{i}\varepsilon)\,f^{1m}\Big)-\frac{\pi^2}{3}\Big]+\mathcal{O}(\epsilon). \quad (1)$$

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References

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