**Divergent Box Integral 4:** $I_4^{D=4-2\epsilon}(0,0,p_3^2,p_4^2;s_{12},s_{23};0,0,0,0)$

Page contributed by R.K. Ellis

The result for the box (see figure) in the unphysical region ($s_{12}<0, s_{23}<0, p_2^2<0, p_4^2<0$) is[1]

$$
\begin{aligned}
& I_4^{D=4-2\epsilon}(0,0,p_3^2,p_4^2;s_{12},s_{23};0,0,0,0) = \frac{\mu^{2\epsilon}}{s_{12}s_{23}} \\
\times\ & \Big[\frac{2}{\epsilon^2}\Big((-s_{12})^{-\epsilon}+(-s_{23})^{-\epsilon}-(-p_3^2)^{-\epsilon}-(-p_4^2)^{-\epsilon}\Big)+\frac{1}{\epsilon^2}\Big((-p_3^2)^{-\epsilon}(-p_4^2)^{-\epsilon}\Big)/(-s_{12})^{-\epsilon} \\
-\ & 2\,\mathrm{Li}_2\left(1-\frac{p_3^2}{s_{23}}\right)-2\,\mathrm{Li}_2\left(1-\frac{p_4^2}{s_{23}}\right)-\ln^2\left(\frac{-s_{12}}{-s_{23}}\right)\Big]+\mathcal{O}(\epsilon)\,.
\end{aligned}
$$

In ref. [1] the auxiliary box function for two adjacent external off-shell lines, is introduced, related to the six dimensional box,

$$
\begin{aligned}
\mathrm{Ls}_{-1}^{2\mathrm{m}h}(s_{12},s_{23};p_3^2,p_4^2) \ &= -\mathrm{Li}_2\left(1-\tfrac{p_3^2}{s_{23}}\right)-\mathrm{Li}_2\left(1-\tfrac{p_4^2}{s_{23}}\right)-\tfrac{1}{2}\ln^2\left(\tfrac{-s_{12}}{-s_{23}}\right)+\tfrac{1}{2}\ln\left(\tfrac{-s_{12}}{-p_3^2}\right)\ln\left(\tfrac{-s_{12}}{-p_4^2}\right) \\
&+\left[\tfrac{1}{2}(s_{12}-p_3^2-p_4^2)+\tfrac{p_3^2p_4^2}{s_{23}}\right]I_3^{3\mathrm{m}}(s_{12},p_3^2,p_4^2)\,,
\end{aligned}
\tag{1}
$$

where $I_3^{3\mathrm{m}}$ is the scalar triangle integral with three lines off shell. This integral vanishes in the appropriate 'back-to-back' kinematic limit. They also introduce a version of this box function with $I_3^{3\mathrm{m}}$ removed,

$$
\widetilde{\mathrm{Ls}}_{-1}^{2\mathrm{m}h}(s_{12},s_{23};p_3^2,p_4^2) = -\mathrm{Li}_2\left(1-\frac{p_3^2}{s_{23}}\right)-\mathrm{Li}_2\left(1-\frac{p_4^2}{s_{23}}\right)-\frac{1}{2}\ln^2\left(\frac{-s_{12}}{-s_{23}}\right)+\frac{1}{2}\ln\left(\frac{-s_{12}}{-p_3^2}\right)\ln\left(\frac{-s_{12}}{-p_4^2}\right)\,. \tag{2}
$$

Thus we may write

$$
\begin{aligned}
& I_4^{D=4-2\epsilon}(0,0,p_3^2,p_4^2;s_{12},s_{23};0,0,0,0) = \frac{\mu^{2\epsilon}}{s_{12}s_{23}} \\
\times\ & \Big[\frac{2}{\epsilon^2}\Big((-s_{12})^{-\epsilon}+(-s_{23})^{-\epsilon}-(-p_3^2)^{-\epsilon}-(-p_4^2)^{-\epsilon}\Big)+\frac{1}{\epsilon^2}\Big((-p_3^2)^{-\epsilon}(-p_4^2)^{-\epsilon}\Big)/(-s_{12})^{-\epsilon} \\
-\ & \ln\left(\frac{-s_{12}}{-p_3^2}\right)\ln\left(\frac{-s_{12}}{-p_4^2}\right)+2\,\widetilde{\mathrm{Ls}}_{-1}^{2\mathrm{m}h}(s_{12},s_{23};p_3^2,p_4^2)\Big]+\mathcal{O}(\epsilon)\,.
\end{aligned}
$$

The result for the six dimensional box is

$$I_4^{D=6}(0,0,p_3^2,p_4^2;s_{12},s_{23};0,0,0,0) = \frac{s_{23}\,\mathrm{Ls}_{-1}^{2\mathrm{m}h}(s_{12},s_{23};p_3^2,p_4^2)}{p_3^2 p_4^2 - s_{23}s_{13}}\,.$$

See the file on notation.

Return to general page on boxes

## References


[1] Z. Bern, L. J. Dixon and D. A. Kosower, Nucl. Phys. B **412**, 751 (1994) [arXiv:hep-ph/9306240]

[2] G. Duplancic and B. Nizic, Eur. Phys. J. C **20**, 357 (2001) [arXiv:hep-ph/0006249]