Divergent Box Integral 5: $I_4^{D=4-2\epsilon}(0,p_2^2,p_3^2,p_4^2;s_{12},s_{23};0,0,0,0)$

Page contributed by R.K. Ellis

The result for the box integral (see figure) with three off-shell legs is [**?**]

$$
\begin{aligned}
& I_4^{D=4-2\epsilon}(0,p_2^2,p_3^2,p_4^2;s_{12},s_{23};0,0,0,0) = \frac{\mu^{2\epsilon}}{(s_{23}s_{12}-p_2^2p_4^2)} \\
\times\ & \left[\frac{2}{\epsilon^2}\Big((-s_{12})^{-\epsilon}+(-s_{23})^{-\epsilon}-(-p_2^2)^{-\epsilon}-(-p_3^2)^{-\epsilon}-(-p_4^2)^{-\epsilon}\Big)\right. \\
+\ & \frac{1}{\epsilon^2}\Big((-p_2^2)^{-\epsilon}(-p_3^2)^{-\epsilon}\Big)/(-s_{23})^{-\epsilon}+\frac{1}{\epsilon^2}\Big((-p_3^2)^{-\epsilon}(-p_4^2)^{-\epsilon}\Big)/(-s_{12})^{-\epsilon} \\
-\ & \left. 2\,\mathrm{Li}_2\left(1-\frac{p_2^2}{s_{12}}\right)-2\,\mathrm{Li}_2\left(1-\frac{p_4^2}{s_{23}}\right)+2\,\mathrm{Li}_2\left(1-\frac{p_2^2p_4^2}{s_{12}s_{23}}\right)-\ln^2\left(\frac{-s_{12}}{-s_{23}}\right)\right]+\mathcal{O}(\epsilon)\,.
\end{aligned}
$$

Explicit instructions on the continuation are given in ref. [**?**].

An alternative result[**?**] deals with the continuation in a more graceful way. We introduce the notation

$$
f^{3m}=\frac{s+t-m_2^2-m_4^2}{st-m_2^2m_4^2}\,,
$$

The three-mass scalar box integral in this expression is

$$
\begin{aligned}
& I_4^{D=4-2\epsilon}(0,p_2^2,p_3^2,p_4^2;s_{12},s_{23};0,0,0,0) = \frac{\mu^{2\epsilon}}{(s_{23}s_{12}-p_2^2p_4^2)} \\
\times\ & \left[\frac{2}{\epsilon^2}\Big((-s_{12}-i\varepsilon)^{-\epsilon}+(-s_{23}-i\varepsilon)^{-\epsilon}-(-p_2^2-i\varepsilon)^{-\epsilon}-(-p_3^2-i\varepsilon)^{-\epsilon}-(-p_4^2-i\varepsilon)^{-\epsilon}\right)\right. \\
+\ & \frac{1}{\epsilon^2}\Big((-p_2^2-i\varepsilon)^{-\epsilon}(-p_3^2-i\varepsilon)^{-\epsilon}\Big)/(-s_{23}-i\varepsilon)^{-\epsilon}+\frac{1}{\epsilon^2}\Big((-p_3^2-i\varepsilon)^{-\epsilon}(-p_4^2-i\varepsilon)^{-\epsilon}\Big)/(-s_{12}-i\varepsilon)^{-\epsilon} \\
& +2\,\mathrm{Li}_2\left(1-\frac{m_2^2+\mathrm{i}\varepsilon}{s_{23}+\mathrm{i}\varepsilon}\right)+2\,\mathrm{Li}_2\left(1-\frac{m_4^2+\mathrm{i}\varepsilon}{s_{12}+\mathrm{i}\varepsilon}\right) \\
& +2\,\mathrm{Li}_2\Big[1-(s_{12}+\mathrm{i}\varepsilon)\,f^{3m}\Big]+2\,\mathrm{Li}_2\Big[1-(s_{23}+\mathrm{i}\varepsilon)\,f^{3m}\Big] \\
& -2\,\mathrm{Li}_2\Big[1-(m_2^2+\mathrm{i}\varepsilon)\,f^{3m}\Big] \\
& \left.-2\,\mathrm{Li}_2\Big[1-(m_4^2+\mathrm{i}\varepsilon)\,f^{3m}\Big]\right\}+\mathcal{O}(\epsilon)\,,
\end{aligned}
$$

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