$I_2^{\{D=4-2\epsilon\}}(s; m_1^2, m_2^2)$

Page contributed by R.K. Ellis

Following 't Hooft and Veltman[1] the result for this bubble integral in our notation is,

$$
\begin{aligned}
I_2^{\{D=4-2\epsilon\}}(s; m_1^2, m_2^2) &= \mu^{2\epsilon}\frac{\Gamma(\epsilon)}{r_\Gamma}\int_0^1 d\lambda\,[-\lambda(1-\lambda)s + \lambda m_2^2 + (1-\lambda)m_1^2 - i\varepsilon]^{-\epsilon} \\
&= \mu^{2\epsilon}\frac{\Gamma(1+\epsilon)}{r_\Gamma}\left[\frac{1}{\epsilon} - \int_0^1 d\lambda\ \ln\big(-\lambda(1-\lambda)s + \lambda m_2^2 + (1-\lambda)m_1^2 - i\varepsilon\big)\right] + \mathcal{O}(\epsilon) \\
&= \mu^{2\epsilon}\frac{\Gamma(1+\epsilon)}{r_\Gamma}\left[\frac{1}{\epsilon} - \ln(s-i\varepsilon) - \int_0^1 d\lambda\ \ln\left(\lambda^2 - \lambda(1 - \frac{m_2^2}{s} + \frac{m_1^2}{s}) + \frac{m_1^2}{s} - \frac{i\varepsilon}{s}\right)\right] + \mathcal{O}(\epsilon) \\
&= \mu^{2\epsilon}\frac{\Gamma(1+\epsilon)}{r_\Gamma}\left[\frac{1}{\epsilon} + 2 - \ln(s-i\varepsilon) + \sum_{i=1}^{2}\left[\lambda_i \ln\left(\frac{\lambda_i - 1}{\lambda_i}\right) - \ln(\lambda_i - 1)\right]\right] + \mathcal{O}(\epsilon)
\end{aligned}
$$

where λ_i are the roots of the quadratic equation,

$$
\lambda_i = \frac{s - m_2^2 + m_1^2 \pm \sqrt{(s - m_2^2 + m_1^2)^2 - 4s(m_1^2 - i\varepsilon)}}{2s}
$$

In deriving this result we have used that

$$
\begin{aligned}
\ln(AB) &= \ln(A) + \ln(B) \quad \text{if imaginary parts of } A \text{ and } B \text{ have opposite signs} \\
\ln(AB + i\varepsilon) &= \ln(A + i\varepsilon) + \ln(B + i\varepsilon/A) \quad \text{for real } A \text{ and } B
\end{aligned}
$$

Results for cases in which some of the parameters take on special values are given below.

1. $I_2^{\{D=4-2\epsilon\}}(s;0,0)$
2. $I_2^{\{D=4-2\epsilon\}}(m^2;0,m^2)$
3. $I_2^{\{D=4-2\epsilon\}}(s;0,m^2)$
4. $I_2^{\{D=4\}}(s;m^2,m^2)$

Return to general page on bubbles

References

[1] G. 't Hooft and M. J. G. Veltman, Nucl. Phys. B **153**, 365 (1979). Inspire