

### 6.3 Skalare Integrale

In diesem Anhang werden alle entweder ultraviolett oder infrarot-kollinear divergenten generischen skalaren Integrale in analytischer Form angegeben. Die Integrale sind mit Hilfe von Feynman-Parametrisierung berechnet und mit Rechnungen von W. Beenakker verglichen, in denen Cutkosky-Schnittregeln und Dispersionsrelationen benutzt werden. Es wird jeweils die kürzere analytische Form angegeben.

Die nicht angegebenen endlichen 3- und 4-Punkt-Integrale sind zum großen Teil berechnet worden oder der Literatur entnommen [56, 52, 53]. Sie können wegen ihrer Länge aber hier nicht dargestellt werden.

Durch die Interferenz mit dem Born-Matrixelement ist nur der Realteil der Integrale von Bedeutung. Durch analytische Fortsetzung von Produkten von Logarithmen können wohldefinierte  $\pi^2$ -Terme auftreten. Als Abkürzungen werden

$$\begin{aligned}
C_\epsilon &= \frac{1}{16\pi^2} e^{-\epsilon\gamma_E} \left( \frac{4\pi\mu^2}{m_{\bar{q}}^2} \right)^\epsilon & (6.44) \\
\beta_{\bar{q}} &= \sqrt{1 - 4m_{\bar{q}}^2/(s + i\epsilon)} & x_{\bar{q}} = \frac{1 - \beta_{\bar{q}}}{1 + \beta_{\bar{q}}} \\
\beta_s &= \sqrt{1 - \frac{4m_{\bar{q}}m_{\bar{g}}}{s - (m_{\bar{q}} - m_{\bar{g}})^2 + i\epsilon}} & x_s = \frac{1 - \beta_s}{1 + \beta_s} \\
\kappa &= \sqrt{(s - m_{\bar{q}}^2 - m_{\bar{g}}^2)^2 - 4m_{\bar{q}}^2m_{\bar{g}}^2 + i\epsilon} \\
t_1 &= t - m_{\bar{q}}^2 & t_g &= t - m_{\bar{g}}^2 \\
u_1 &= u - m_{\bar{q}}^2 & u_g &= u - m_{\bar{g}}^2 \\
m_-^2 &= m_{\bar{g}}^2 - m_{\bar{q}}^2 - i\epsilon
\end{aligned}$$

verwendet.

Das skalare 1-Punkt-Integral:

$$A(m) = \mu^{2\epsilon} \int \frac{d^n q}{(2\pi)^n} \frac{1}{q^2 - m^2} = iC_\epsilon m^2 \left[ \frac{1}{\epsilon} + 1 - \log \left( \frac{m^2}{m_{\bar{q}}^2} \right) \right] \quad (6.45)$$

Die skalaren 2-Punkt-Integrale:

$$B_0(p^2; m_1, m_2) = \mu^{2\epsilon} \int \frac{d^n q}{(2\pi)^n} \frac{1}{[q^2 - m_1^2][(q+p)^2 - m_2^2]} \quad (6.46)$$

$$B_0(0; 0, 0) = 0 \quad (6.47)$$

$$B_0(0; m, m) = iC_\epsilon \left[ \frac{1}{\epsilon} - \log \left( \frac{m^2}{m_{\bar{q}}^2} \right) \right] \quad (6.48)$$

$$B_0(0; m_1, m_2) = iC_\epsilon \left[ \frac{1}{\epsilon} + 1 - \frac{m_1^2}{m_1^2 - m_2^2} \log \left( \frac{m_1^2}{m_{\bar{q}}^2} \right) - \frac{m_2^2}{m_2^2 - m_1^2} \log \left( \frac{m_2^2}{m_{\bar{q}}^2} \right) \right] \quad (6.49)$$

$$B_0(s; 0, 0) = iC_\varepsilon \left[ \frac{1}{\varepsilon} + 2 - \log \left( \frac{s}{m_q^2} \right) \right] \quad (6.50)$$

$$B_0(m^2; m, 0) = iC_\varepsilon \left[ \frac{1}{\varepsilon} + 2 - \log \left( \frac{m^2}{m_q^2} \right) \right] \quad (6.51)$$

$$B_0(q^2; m_1, 0) = iC_\varepsilon \left[ \frac{1}{\varepsilon} + 2 - \frac{q^2 - m_1^2}{q^2} \log \left( \frac{m_1^2 - q^2}{m_1^2} \right) - \log \left( \frac{m_1^2}{m_q^2} \right) \right] \quad (6.52)$$

$$B_0(q^2; m_1, m_2) = iC_\varepsilon \left[ \frac{1}{\varepsilon} + 2 + x_+ \log(1 - 1/x_+) + x_- \log(1 - 1/x_-) - \log \left( \frac{m_1^2}{m_q^2} \right) \right] \quad (6.53)$$

$$x_\pm = \frac{q^2 + m_2^2 - m_1^2}{2(q^2 + i\epsilon)} \pm \sqrt{\left( \frac{q^2 + m_2^2 - m_1^2}{2(q^2 + i\epsilon)} \right)^2 - \frac{m_2^2}{q^2 + i\epsilon}}$$

Die Ableitung der skalaren 2-Punkt-Integrale:

$$B'_0(p^2; m_1, m_2) = \left. \frac{\partial}{\partial q^2} B_0(q^2; m_1, m_2) \right|_{q^2=p^2} \quad (6.54)$$

$$B'_0(0; 0, 0) = 0 \quad (6.55)$$

$$B'_0(0; m, m) = iC_\varepsilon \left[ \frac{1}{6m^2} \right] \quad (6.56)$$

$$B'_0(0; m_1, m_2) = iC_\varepsilon \left[ \frac{m_1^2 + m_2^2}{2} - \frac{m_1^2 m_2^2}{m_1^2 - m_2^2} \log \left( \frac{m_1^2}{m_2^2} \right) \right] \frac{1}{(m_1^2 - m_2^2)^2} \quad (6.57)$$

$$B'_0(m^2; m, 0) = iC_\varepsilon \left[ -\frac{1}{2\varepsilon} - 1 + \frac{1}{2} \log \left( \frac{m^2}{m_q^2} \right) \right] \frac{1}{m^2} \quad (6.58)$$

$$B'_0(m^2; m_1, 0) = iC_\varepsilon \left[ -1 - \frac{m_1^2}{m^2} \log \left( \frac{m_1^2 - m^2}{m_1^2} \right) \right] \frac{1}{m^2} \quad (6.59)$$

$$B'_0(q^2; m_1, m_2) = iC_\varepsilon \left[ -\frac{1}{q^2} + \frac{(x_+ - x_+^2) \log(1 - 1/x_+) - (x_- - x_-^2) \log(1 - 1/x_-)}{q^2 (x_+ - x_-)} \right] \quad (6.60)$$

$$x_\pm = \frac{q^2 + m_2^2 - m_1^2}{2(q^2 + i\epsilon)} \pm \sqrt{\left( \frac{q^2 + m_2^2 - m_1^2}{2(q^2 + i\epsilon)} \right)^2 - \frac{m_2^2}{q^2 + i\epsilon}}$$

Die skalaren 3-Punkt-Integrale:

$$\begin{aligned} C_0(p_1^2, p_2^2, (p_1 + p_2)^2; m_1, m_2, m_3) &= \quad (6.61) \\ &= \mu^{2\varepsilon} \int \frac{d^n q}{(2\pi)^n} \frac{1}{[q^2 - m_1^2][(q + p_1)^2 - m_2^2][(q + p_1 + p_2)^2 - m_3^2]} \end{aligned}$$

Im  $s$ -Kanal mit einlaufenden masselosen Partonen:

$$C_0(0, 0, s; 0, 0, 0) = iC_\varepsilon \frac{1}{s} \left[ \frac{1}{\varepsilon^2} - \frac{1}{\varepsilon} \log \left( \frac{s}{m_q^2} \right) + \frac{1}{2} \log^2 \left( \frac{s}{m_q^2} \right) - \frac{7}{12} \pi^2 \right] \quad (6.62)$$

Im  $s$ -Kanal mit auslaufenden massiven Squarks:

$$C_0(m_{\bar{q}}^2, m_{\bar{q}}^2, s; m_{\bar{q}}, 0, m_{\bar{q}}) = \quad (6.63)$$

$$= iC_\varepsilon \frac{1}{s\beta_{\bar{q}}} \left[ \frac{1}{\varepsilon} \log(x_{\bar{q}}) - 2 \log(x_{\bar{q}}) \log(1 - x_{\bar{q}}) - 2 \text{Li}_2(x_{\bar{q}}) + \frac{1}{2} \log^2(x_{\bar{q}}) - \frac{2}{3} \pi^2 \right]$$

Im  $s$ -Kanal mit auslaufenden massiven Squarks und Gluinos:

$$C_0(m_{\bar{q}}^2, m_{\bar{g}}^2, s; m_{\bar{q}}, 0, m_{\bar{g}}) = \quad (6.64)$$

$$= iC_\varepsilon \frac{1}{\kappa} \left[ \log(-x_s) \left( \frac{1}{\varepsilon} - \log\left(\frac{m_{\bar{g}}}{m_{\bar{q}}}\right) + \frac{1}{2} \log(-x_s) - 2 \log(1 - x_s^2) \right) + \frac{1}{6} \pi^2 \right. \\ \left. - \frac{1}{2} \log^2\left(\frac{m_{\bar{q}}}{m_{\bar{g}}}\right) - \text{Li}_2(x_s^2) - \text{Li}_2\left(1 + \frac{m_{\bar{g}} x_s}{m_{\bar{q}}}\right) - \text{Li}_2\left(1 + \frac{m_{\bar{q}} x_s}{m_{\bar{g}}}\right) \right]$$

Im  $t$ -Kanal mit einem masselosen Parton und einem massiven Squark:

$$C_0(0, m_{\bar{q}}^2, t; 0, 0, m_{\bar{q}}) = \quad (6.65)$$

$$= iC_\varepsilon \frac{1}{t_1} \left[ \frac{1}{2\varepsilon^2} - \frac{1}{\varepsilon} \log\left(\frac{-t_1}{m_{\bar{q}}^2}\right) + \log^2\left(\frac{-t_1}{m_{\bar{q}}^2}\right) + \text{Li}_2\left(\frac{t}{m_{\bar{q}}^2}\right) + \frac{1}{24} \pi^2 \right]$$

$$C_0(0, m_{\bar{q}}^2, t; 0, 0, m_{\bar{g}}) = \quad (6.66)$$

$$= iC_\varepsilon \frac{1}{t_1} \left[ -\frac{1}{\varepsilon} \log\left(\frac{-t_g}{m_{\bar{g}}^2 - m_{\bar{q}}^2}\right) + \text{Li}_2\left(\frac{t}{m_{\bar{g}}^2}\right) - \text{Li}_2\left(\frac{m_{\bar{q}}^2}{m_{\bar{g}}^2}\right) + \log^2\left(1 - \frac{t}{m_{\bar{g}}^2}\right) \right. \\ \left. - \log^2\left(1 - \frac{m_{\bar{q}}^2}{m_{\bar{g}}^2}\right) + \log\left(\frac{m_{\bar{g}}^2}{m_{\bar{q}}^2}\right) \log\left(\frac{-t_g}{m_{\bar{g}}^2 - m_{\bar{q}}^2}\right) \right]$$

Die skalaren 4-Punkt-Integrale:

$$D_0(p_1^2, p_2^2, p_3^2, (p_1 + p_2 + p_3)^2, (p_1 + p_2)^2, (p_2 + p_3)^2; m_1, m_2, m_3, m_4) = \quad (6.67)$$

$$= \mu^{2\varepsilon} \int \frac{d^n q}{(2\pi)^n} \frac{1}{[q^2 - m_1^2][(q + p_1)^2 - m_2^2][(q + p_1 + p_2)^2 - m_3^2][(q + p_1 + p_2 + p_3)^2 - m_4^2]}$$

Die skalaren 4-Punkt-Integrale für zwei massive Squarks:

$$D_0(0, 0, m_{\bar{q}}^2, m_{\bar{q}}^2, s, t; 0, 0, 0, m_{\bar{q}}) = \quad (6.68)$$

$$= iC_\varepsilon \frac{1}{st_1} \left[ \frac{2}{\varepsilon^2} - \frac{1}{\varepsilon} \left( 2 \log\left(\frac{-t_1}{m_{\bar{q}}^2}\right) + \log\left(\frac{s}{m_{\bar{q}}^2}\right) \right) + 2 \log\left(\frac{-t_1}{m_{\bar{q}}^2}\right) \log\left(\frac{s}{m_{\bar{q}}^2}\right) - \frac{2}{3} \pi^2 \right]$$

$$D_0(0, 0, m_{\bar{q}}^2, m_{\bar{q}}^2, s, t; m_{\bar{q}}, m_{\bar{q}}, m_{\bar{q}}, 0) = \quad (6.69)$$

$$= iC_\varepsilon \frac{1}{st_1 \beta_{\bar{q}}} \left[ \frac{1}{\varepsilon} \log(x_{\bar{q}}) - 2 \log(x_{\bar{q}}) \log(1 - x_{\bar{q}}) + 2 \log(x_{\bar{q}}) \log(1 + x_{\bar{q}}) \right. \\ \left. - 2 \log(x_{\bar{q}}) \log\left(\frac{-t_1}{m_{\bar{q}}^2}\right) - 2 \operatorname{Li}_2(x_{\bar{q}}) + 2 \operatorname{Li}_2(-x_{\bar{q}}) - \frac{1}{2} \pi^2 \right]$$

$$D_0(0, m_{\bar{q}}^2, 0, m_{\bar{q}}^2, t, u; m_{\bar{q}}, m_{\bar{q}}, 0, 0) = \quad (6.70)$$

$$= iC_\varepsilon \frac{1}{t_1 u_1} \left[ \frac{1}{\varepsilon^2} - \frac{1}{\varepsilon} \left( \log\left(\frac{-t_1}{m_{\bar{q}}^2}\right) + \log\left(\frac{-u_1}{m_{\bar{q}}^2}\right) \right) + 2 \log\left(\frac{-t_1}{m_{\bar{q}}^2}\right) \log\left(\frac{-u_1}{m_{\bar{q}}^2}\right) - \frac{7}{12} \pi^2 \right]$$

$$D_0(0, 0, m_{\bar{q}}^2, m_{\bar{q}}^2, s, t; 0, 0, 0, m_{\bar{g}}) = \quad (6.71)$$

$$= iC_\varepsilon \frac{1}{st_g} \left[ \frac{1}{\varepsilon^2} - \frac{1}{\varepsilon} \left( \log\left(\frac{s}{m_{\bar{q}}^2}\right) + 2 \log\left(\frac{-t_g}{m_{\bar{q}}^2}\right) \right) - 4 \operatorname{Li}_2\left(1 + \frac{m_{\bar{q}}^2}{t_g}\right) - \frac{1}{4} \pi^2 \right. \\ \left. - \operatorname{Li}_2\left(1 + \frac{m_{\bar{q}}^4}{sm_{\bar{g}}^2}\right) + \frac{1}{2} \log^2\left(\frac{s}{m_{\bar{q}}^2}\right) - \frac{1}{2} \log^2\left(\frac{s}{m_{\bar{g}}^2}\right) + 2 \log\left(\frac{s}{m_{\bar{q}}^2}\right) \log\left(\frac{-t_g}{m_{\bar{g}}^2}\right) \right. \\ \left. - 2 \log\left(\frac{m_{\bar{q}}^2}{m_{\bar{g}}^2}\right) \log\left(\frac{m_{\bar{q}}^2}{m_{\bar{g}}^2}\right) \right]$$

$$D_0(0, m_{\bar{q}}^2, 0, m_{\bar{q}}^2, t, u; m_{\bar{g}}, m_{\bar{g}}, 0, 0) = \quad (6.72)$$

$$= iC_\varepsilon \frac{1}{t_g u_g - m_{\bar{q}}^4} \left[ \left( \frac{1}{\varepsilon} - \log\left(\frac{m_{\bar{q}}^4}{m_{\bar{q}}^2 m_{\bar{g}}^2}\right) \right) \log\left(\frac{m_{\bar{q}}^4}{t_g u_g}\right) - \log^2\left(\frac{m_{\bar{q}}^2}{-t_g}\right) - \log^2\left(\frac{m_{\bar{q}}^2}{-u_g}\right) \right. \\ \left. - 4 \operatorname{Li}_2\left(1 + \frac{m_{\bar{q}}^2}{t_g}\right) - 4 \operatorname{Li}_2\left(1 + \frac{m_{\bar{q}}^2}{u_g}\right) - 2 \operatorname{Li}_2\left(1 - \frac{t_g u_g}{m_{\bar{q}}^4}\right) \right]$$

$$D_0(0, 0, m_{\bar{q}}^2, m_{\bar{q}}^2, s, t; m_{\bar{q}}, m_{\bar{g}}, m_{\bar{q}}, 0) = \quad (6.73)$$

$$= iC_\varepsilon \frac{1}{st_g \beta_{\bar{q}}} \left[ \frac{1}{\varepsilon} \log(x_{\bar{q}}) - 2 \operatorname{Li}_2(x_{\bar{q}}) - \operatorname{Li}_2\left(1 + \frac{m_{\bar{g}}^2 x_{\bar{q}}}{m_{\bar{q}}^2}\right) - \operatorname{Li}_2\left(1 + \frac{m_{\bar{q}}^2 x_{\bar{q}}}{m_{\bar{g}}^2}\right) \right. \\ \left. - \frac{1}{6} \pi^2 - \frac{1}{2} \log^2\left(\frac{m_{\bar{g}}^2}{m_{\bar{q}}^2}\right) - 2 \log(x_{\bar{q}}) \left( \log(1 - x_{\bar{q}}) + \log\left(\frac{-t_g}{m_{\bar{q}} m_{\bar{g}}}\right) \right) \right]$$

$$D_0(0, m_{\bar{q}}^2, 0, m_{\bar{q}}^2, t, u; m_{\bar{g}}, m_{\bar{q}}, 0, 0) = \quad (6.74)$$

$$= iC_\varepsilon \frac{1}{t_g u_1} \left[ \frac{1}{2\varepsilon^2} - \frac{1}{\varepsilon} \left( \log\left(\frac{-t_g}{m_{\bar{q}}^2}\right) + \log\left(\frac{-u_1}{m_{\bar{q}}^2}\right) \right) + \log^2\left(\frac{-u_1}{m_{\bar{q}}^2}\right) - \frac{1}{8} \pi^2 \right. \\ \left. + 2 \log\left(\frac{-u_1}{m_{\bar{q}}^2}\right) \log\left(\frac{-t_g}{m_{\bar{q}}^2}\right) + \operatorname{Li}_2\left(1 + \frac{u_1}{m_{\bar{q}}^2}\right) + \operatorname{Li}_2\left(1 + \frac{m_{\bar{g}}^2 u_1}{m_{\bar{q}}^2 m_{\bar{q}}^2}\right) - 2 \operatorname{Li}_2\left(1 + \frac{m_{\bar{q}}^2}{t_g}\right) \right]$$

Die skalaren 4-Punkt-Integrale für je ein massives Squark und Gluino:

$$D_0(0, 0, m_2^2, m_1^2, s, t; 0, 0, 0, m_1) = \quad (6.75)$$

$$= iC_\epsilon \frac{1}{s(t-m_1^2)} \left[ \frac{3}{2\epsilon^2} - \frac{1}{\epsilon} \left( 2 \log \left( \frac{m_1^2-t}{m_1 m_{\bar{q}}} \right) + \log \left( \frac{s}{m_{\bar{q}}^2} \right) - \log \left( \frac{m_1^2-m_2^2}{m_1 m_{\bar{q}}} \right) \right) \right. \\ \left. - \frac{13}{24}\pi^2 - \log^2 \left( \frac{m_1^2-m_2^2}{m_1 m_{\bar{q}}} \right) + 2 \log \left( \frac{m_1^2-t}{m_1 m_{\bar{q}}} \right) \log \left( \frac{s}{m_{\bar{q}}^2} \right) - 2 \text{Li}_2 \left( \frac{m_2^2-t}{m_1^2-t} \right) \right]$$

$$D_0(0, 0, m_1^2, m_2^2, s, t; m_2, m_1, m_1, 0) = \quad (6.76)$$

$$= iC_\epsilon \frac{1}{\kappa(t-m_1^2)} \left[ \log(x_s) \left( \frac{1}{\epsilon} - 2 \log \left( \frac{m_1^2-t}{m_1 m_{\bar{q}}} \right) - 2 \log(1-x_s^2) \right) + \frac{1}{6}\pi^2 \right. \\ \left. - \log^2 \left( \frac{m_1}{m_2} \right) - \text{Li}_2(x_s^2) - 2 \text{Li}_2 \left( 1 + \frac{m_1 x_s}{m_2} \right) - 2 \text{Li}_2 \left( 1 + \frac{m_2 x_s}{m_1} \right) \right]$$

$$D_0(0, m_1^2, 0, m_2^2, t, u; 0, 0, m_1, m_2) = \quad (6.77)$$

$$= iC_\epsilon \frac{1}{(t-m_1^2)(u-m_2^2)} \left[ \frac{1}{\epsilon^2} - \frac{1}{\epsilon} \left( \log \left( \frac{m_2^2-u}{m_2 m_{\bar{q}}} \right) + \log \left( \frac{m_1^2-t}{m_1 m_{\bar{q}}} \right) \right) - \frac{7}{12}\pi^2 \right. \\ \left. - \log^2 \left( \frac{m_1}{m_2} \right) + 2 \log \left( \frac{m_2^2-u}{m_2 m_{\bar{q}}} \right) \log \left( \frac{m_1^2-t}{m_1 m_{\bar{q}}} \right) \right]$$

$$D_0(0, m_1^2, 0, m_2^2, t, u; 0, 0, m_1, m_1) = \quad (6.78)$$

$$= iC_\epsilon \frac{1}{(t-m_1^2)(u-m_1^2)} \left[ \frac{1}{2\epsilon^2} - \frac{1}{\epsilon} \left( \log \left( \frac{m_1^2-u}{m_1^2-m_2^2} \right) + \log \left( \frac{m_1^2-t}{m_1 m_{\bar{q}}} \right) \right) - \frac{1}{8}\pi^2 \right. \\ \left. + \log^2 \left( \frac{m_1^2-t}{m_1 m_{\bar{q}}} \right) + 2 \log \left( \frac{m_1^2-t}{m_1 m_{\bar{q}}} \right) \log \left( \frac{m_1^2-u}{m_1^2-m_2^2} \right) \right. \\ \left. + 2 \text{Li}_2 \left( \frac{t-m_2^2}{m_1^2-m_2^2} \right) - 2 \text{Li}_2 \left( \frac{m_2^2-u}{m_1^2-u} \right) \right]$$

$$D_0(0, m_1^2, 0, m_2^2, t, u; 0, 0, m_2, m_1) = \quad (6.79)$$

$$= iC_\epsilon \frac{1}{(t-m_2^2)(u-m_1^2) + (m_1^2-m_2^2)^2} \left[ -\frac{1}{\epsilon} \left( \log \left( \frac{m_2^2-t}{m_2^2-m_1^2} \right) + \log \left( \frac{m_1^2-u}{m_1^2-m_2^2} \right) \right) \right. \\ \left. + 2 \log \left( \frac{m_2^2-t}{m_2 m_{\bar{q}}} \right) \log \left( \frac{m_1^2-u}{m_1 m_{\bar{q}}} \right) - \log^2 \left( \frac{m_2^2-m_1^2}{m_2 m_{\bar{q}}} \right) - \log^2 \left( \frac{m_1^2-m_2^2}{m_1 m_{\bar{q}}} \right) \right. \\ \left. - \log^2 \left( \frac{m_1}{m_2} \right) - 2 \text{Li}_2 \left( \frac{m_1^2-t}{m_2^2-t} \right) - 2 \text{Li}_2 \left( \frac{m_2^2-u}{m_1^2-u} \right) \right. \\ \left. - \text{Li}_2 \left( 1 - \frac{m_1^2-m_2^2}{m_2^2-t} \right) - \text{Li}_2 \left( 1 - \frac{m_2^2-m_1^2}{m_1^2-u} \right) - \text{Li}_2 \left( 1 - \frac{m_2^2}{m_1^2} \frac{m_1^2-m_2^2}{m_2^2-t} \right) \right. \\ \left. - \text{Li}_2 \left( 1 - \frac{m_1^2}{m_2^2} \frac{m_2^2-m_1^2}{m_1^2-u} \right) + 2 \text{Li}_2 \left( 1 - \frac{m_2^2-m_1^2}{m_1^2-u} \frac{m_1^2-m_2^2}{m_2^2-t} \right) \right]$$