

Notation

We work in the Bjorken-Drell metric so that $l^2 = l_0^2 - l_1^2 - l_2^2 - l_3^2$. The definition of the integrals are as follows

$$I_4^{\{D\}}(p_1^2, p_2^2, p_3^2, p_4^2; (p_1 + p_2)^2, (p_2 + p_3)^2; m_1^2, m_2^2, m_3^2, m_4^2) = \frac{\mu^{4-D}}{i\pi^{\frac{D}{2}} r_\Gamma} \int d^D l \frac{1}{(l^2 - m_1^2 + i\varepsilon)((l + q_1)^2 - m_2^2 + i\varepsilon)((l + q_2)^2 - m_3^2 + i\varepsilon)((l + q_3)^2 - m_4^2 + i\varepsilon)}$$

$$I_3^{\{D\}}(p_1^2, p_2^2, p_3^2; m_1^2, m_2^2, m_3^2) = \frac{\mu^{4-D}}{i\pi^{\frac{D}{2}} r_\Gamma} \int d^D l \frac{1}{(l^2 - m_1^2 + i\varepsilon)((l + q_1)^2 - m_2^2 + i\varepsilon)((l + q_2)^2 - m_3^2 + i\varepsilon)}$$

$$I_2^{\{D\}}(p_1^2; m_1^2, m_2^2) = \frac{\mu^{4-D}}{i\pi^{\frac{D}{2}} r_\Gamma} \int d^D l \frac{1}{(l^2 - m_1^2 + i\varepsilon)((l + q_1)^2 - m_2^2 + i\varepsilon)}$$

$$I_2^{\{D\}}(p_1^2; m_1^2, m_2^2) = \frac{\mu^{4-D}}{i\pi^{\frac{D}{2}} r_\Gamma} \int d^D l \frac{1}{(l^2 - m_1^2 + i\varepsilon)}$$

where $q_1 = p_1, q_2 = p_1 + p_2, q_3 = p_1 + p_2 + p_3$ and $q_0 = q_4 = 0$.

Near four dimensions we use $D = 4 - 2\varepsilon$. The symbol ε is thus equal to $\frac{4-D}{2}$. (For clarity the small imaginary part which fixes the analytic continuations is specified by $+i\varepsilon$). The overall constant which occurs in D -dimensional integrals is,

$$r_\Gamma = \frac{\Gamma^2(1 - \varepsilon)\Gamma(1 + \varepsilon)}{\Gamma(1 - 2\varepsilon)} = \frac{1}{\Gamma(1 - \varepsilon)} + \mathcal{O}(\varepsilon^3).$$

Since

$$\begin{aligned} \Gamma(1 - \varepsilon) &= 1 + \varepsilon\gamma + \varepsilon^2 \left[\frac{\gamma^2}{2} + \frac{\pi^2}{12} \right] + \mathcal{O}(\varepsilon^3) \\ r_\Gamma &= 1 - \varepsilon\gamma + \varepsilon^2 \left[\frac{\gamma^2}{2} - \frac{\pi^2}{12} \right] + \mathcal{O}(\varepsilon^3). \end{aligned}$$

Various useful formula for dimensional regularization are given [here](#).

The final results are given in terms of logarithms and dilogarithms. The logarithm is defined to have a cut along the negative real axis. The rule for the logarithm of a product is

$$\begin{aligned}\ln(ab) &= \ln a + \ln b + \eta(a, b) \\ \eta(a, b) &= 2\pi i [\theta(-\text{Im}(a))\theta(-\text{Im}(b))\theta(\text{Im}(ab)) - \theta(\text{Im}(a))\theta(\text{Im}(b))\theta(-\text{Im}(ab))]\end{aligned}$$

So that

$$\begin{aligned}\ln(ab) &= \ln a + \ln b, \quad \text{if } \text{Im}(a) \text{ and } \text{Im}(b) \text{ have different signs.} \\ \ln\left(\frac{a}{b}\right) &= \ln a - \ln b, \quad \text{if } \text{Im}(a) \text{ and } \text{Im}(b) \text{ have the same sign.}\end{aligned}$$

The dilogarithm is defined as

$$\text{Li}_2(x) = - \int_0^x \frac{dx}{x} \ln(1-x) = \frac{x}{1^2} + \frac{x^2}{2^2} + \frac{x^3}{3^2} + \dots \text{ when } |x| \leq 1$$

A number of the most commonly useful dilogarithm identities are given [here](#).

[Back to QCDLoop Home page](#)