Notation

We work in the Bjorken-Drell metric so that $l^2 = l_0^2 - l_1^2 - l_2^2 - l_3^2$. The definition of the integrals are as follows

$$I_4^{\{D\}}(p_1^2, p_2^2, p_3^2, p_4^2; (p_1+p_2)^2, (p_2+p_3)^2; m_1^2, m_2^2, m_3^2, m_4^2) =$$
$$\frac{\mu^{4-D}}{i\pi^{\frac{D}{2}} r_\Gamma} \int d^D l \, \frac{1}{(l^2 - m_1^2 + i\varepsilon)((l+q_1)^2 - m_2^2 + i\varepsilon)((l+q_2)^2 - m_3^2 + i\varepsilon)((l+q_3)^2 - m_4^2 + i\varepsilon)}$$

$$I_3^{\{D\}}(p_1^2, p_2^2, p_3^2; m_1^2, m_2^2, m_3^2) = \frac{\mu^{4-D}}{i\pi^{\frac{D}{2}} r_\Gamma} \int d^D l \, \frac{1}{(l^2 - m_1^2 + i\varepsilon)((l+q_1)^2 - m_2^2 + i\varepsilon)((l+q_2)^2 - m_3^2 + i\varepsilon)}$$

$$I_2^{\{D\}}(p_1^2; m_1^2, m_2^2) = \frac{\mu^{4-D}}{i\pi^{\frac{D}{2}} r_\Gamma} \int d^D l \, \frac{1}{(l^2 - m_1^2 + i\varepsilon)((l+q_1)^2 - m_2^2 + i\varepsilon)}$$

$$I_1^{\{D\}}(m_1^2) = \frac{\mu^{4-D}}{i\pi^{\frac{D}{2}} r_\Gamma} \int d^D l \, \frac{1}{(l^2 - m_1^2 + i\varepsilon)}$$

where $q_1 = p_1, q_2 = p_1 + p_2, q_3 = p_1 + p_2 + p_3$ and $q_0 = q_4 = 0$.

Near four dimensions we use $D = 4 - 2\epsilon$. The symbol ϵ is thus equal to $\frac{4-D}{2}$. (For clarity the small imaginary part which fixes the analytic continuations is specified by $+i\varepsilon$). The overall constant which occurs in D-dimensional integrals is,

$$r_\Gamma = \frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)} = \frac{1}{\Gamma(1-\epsilon)} + \mathcal{O}(\epsilon^3) \, .$$

Since

$$\begin{aligned} \Gamma(1-\epsilon) &= 1 + \epsilon\gamma + \epsilon^2 \left[\frac{\gamma^2}{2} + \frac{\pi^2}{12}\right] + \mathcal{O}(\epsilon^3) \\ r_\Gamma &= 1 - \epsilon\gamma + \epsilon^2 \left[\frac{\gamma^2}{2} - \frac{\pi^2}{12}\right] + \mathcal{O}(\epsilon^3) \, . \end{aligned}$$

Various useful formula for dimensional regularization are given here.

The final results are given in terms of logarithms and dilogarithms. The logarithm is defined to have a cut along the negative real axis. The rule for the logarithm of a product is

$$\begin{aligned} \ln(ab) &= \ln a + \ln b + \eta(a,b) \\ \eta(a,b) &= 2\pi i\big[\theta(-\mathrm{Im}(a))\theta(-\mathrm{Im}(b))\theta(\mathrm{Im}(ab)) - \theta(\mathrm{Im}(a))\theta(\mathrm{Im}(b))\theta(-\mathrm{Im}(ab))\big] \end{aligned}$$

So that

$$\begin{aligned} \ln(ab) &= \ln a + \ln b, \quad \text{if } \mathrm{Im}(a) \text{ and } \mathrm{Im}(b) \text{ have different signs.} \\ \ln(\frac{a}{b}) &= \ln a - \ln b, \quad \text{if } \mathrm{Im}(a) \text{ and } \mathrm{Im}(b) \text{ have the same sign.} \end{aligned}$$

The dilogarithm is defined as

$$\mathrm{Li}_2(x) = -\int_0^x \frac{dx}{x}\ln(1-x) = \frac{x}{1^2} + \frac{x^2}{2^2} + \frac{x^3}{3^2} + \ldots \text{when} |x| \leq 1$$

A number of the most commonly useful dilogarithm identities are given here.

Back to QCDLoop Home page