

## Triangle integrals

Jump directly to divergent triangle integrals

Jump directly to finite triangle integrals

The definition of the triangle integrals are as follows

$$I_3^{\{D\}}(p_1^2, p_2^2, p_3^2; m_1^2, m_2^2, m_3^2) = \frac{\mu^{4-D}}{i\pi^{\frac{D}{2}} r_\Gamma} \int d^D l \frac{1}{(l^2 - m_1^2 + i\varepsilon)((l + q_1)^2 - m_2^2 + i\varepsilon)((l + q_2)^2 - m_3^2 + i\varepsilon)}$$

where  $q_1 = p_1, q_2 = p_1 + p_2$ .

After Feynman parametrization and integration over the loop momentum  $l$  we have

$$I_3^{\{D=4-2\epsilon\}}(p_1^2, p_2^2, p_3^2; m_1^2, m_2^2, m_3^2) = -\mu^{2\epsilon} \frac{\Gamma(1+\epsilon)}{r_\Gamma} \int_0^1 da_1 da_2 da_3 \frac{\delta(1 - a_1 - a_2 - a_3)}{\left[ \sum_{i,j=1}^3 a_i a_j Y_{ij} - i\varepsilon \right]^{1+\epsilon}}$$

$Y$  is the so-called modified Cayley matrix.

$$Y_{ij} = \frac{1}{2} \left[ m_i^2 + m_j^2 - (q_{i-1} - q_{j-1})^2 \right]$$

$$Y = \begin{pmatrix} m_1^2 & \frac{1}{2}m_1^2 + \frac{1}{2}m_2^2 - \frac{1}{2}p_1^2 & \frac{1}{2}m_1^2 + \frac{1}{2}m_3^2 - \frac{1}{2}p_3^2 \\ \frac{1}{2}m_1^2 + \frac{1}{2}m_2^2 - \frac{1}{2}p_1^2 & m_2^2 & \frac{1}{2}m_2^2 + \frac{1}{2}m_3^2 - \frac{1}{2}p_2^2 \\ \frac{1}{2}m_1^2 + \frac{1}{2}m_3^2 - \frac{1}{2}p_3^2 & \frac{1}{2}m_2^2 + \frac{1}{2}m_3^2 - \frac{1}{2}p_2^2 & m_3^2 \end{pmatrix}$$

A necessary condition for a soft singularity is that for a least one value of  $i$

$$Y_{ii} = Y_{i i+1} = Y_{i i-1} = 0$$

A necessary condition for a collinear singularity is that for a least one value of  $i$

$$Y_{ii} = Y_{i+1 i+1} = Y_{i i+1} = 0$$

If neither of these conditions is satisfied the integral is finite and can be evaluated by general techniques[1, 2, 3]

### Massless internal lines (clickable links)

1.  $I_3^{\{D=4-2\epsilon\}}(0, 0, p_3^2; 0, 0, 0)$
2.  $I_3^{\{D=4-2\epsilon\}}(p_1^2, p_2^2, 0; 0, 0, 0)$

### One massive internal line

The three general cases of divergent integrals are

3.  $I_3^{\{D=4-2\epsilon\}}(0, p_2^2, p_3^2; 0, 0, m^2)$
4.  $I_3^{\{D=4-2\epsilon\}}(0, p_2^2, m^2; 0, 0, m^2)$
5.  $I_3^{\{D=4-2\epsilon\}}(0, m^2, m^2; 0, 0, m^2)$

### Two massive internal lines

With two massive internal lines we can only have a soft singularity.  $I_4^{\{D=4-2\epsilon\}}(m_2^2, s_{12}, m_3^2; 0, m_2^2, m_3^2)$  is the most general divergent integral.

6.  $I_3^{\{D=4-2\epsilon\}}(m_2^2, s_{12}, m_3^2; 0, m_2^2, m_3^2)$

[Back to QCDLoop Home page](#)

[Go to page on finite triangles](#)

### References

- [1] G. 't Hooft and M. J. G. Veltman, Nucl. Phys. B **153**, 365 (1979). [Inspire](#)

[2] G. J. van Oldenborgh, Comput. Phys. Commun. **66**, 1 (1991). [Inspire](#)

[3] T. Hahn and M. Perez-Victoria, Comput. Phys. Commun. **118**, 153 (1999) [[arXiv:hep-ph/9807565](#)].  
[\[arXiv:hep-ph/9807565\]](#)