# Triangle integrals

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Jump directly to finite triangle integrals

The definition of the triangle integrals are as follows

$$
\begin{aligned}
& I_3^{\{D\}}(p_1^2, p_2^2, p_3^2; m_1^2, m_2^2, m_3^2) \\
= & \frac{\mu^{4-D}}{i\pi^{\frac{D}{2}} r_\Gamma} \int d^D l \frac{1}{(l^2 - m_1^2 + i\varepsilon)((l+q_1)^2 - m_2^2 + i\varepsilon)((l+q_2)^2 - m_3^2 + i\varepsilon)}
\end{aligned}
$$

where $q_1 = p_1, q_2 = p_1 + p_2$.

After Feynman parametrization and integration over the loop momentum $l$ we have

$$
I_3^{\{D=4-2\epsilon\}}(p_1^2, p_2^2, p_3^2; m_1^2, m_2^2, m_3^2) = -\mu^{2\epsilon} \frac{\Gamma(1+\epsilon)}{r_\Gamma} \int_0^1 \, da_1 da_2 da_3 \, \frac{\delta(1 - a_1 - a_2 - a_3)}{\left[\sum_{i,j=1}^3 a_i a_j Y_{ij} - i\varepsilon\right]^{1+\epsilon}}
$$

$Y$ is the so-called modified Cayley matrix.

$$
Y_{ij} = \frac{1}{2}\Big[m_i^2 + m_j^2 - (q_{i-1} - q_{j-1})^2\Big]
$$

$$
Y = \begin{pmatrix}
m_1^2 & \frac{1}{2}m_1^2 + \frac{1}{2}m_2^2 - \frac{1}{2}p_1^2 & \frac{1}{2}m_1^2 + \frac{1}{2}m_3^2 - \frac{1}{2}p_3^2 \\
\frac{1}{2}m_1^2 + \frac{1}{2}m_2^2 - \frac{1}{2}p_1^2 & m_2^2 & \frac{1}{2}m_2^2 + \frac{1}{2}m_3^2 - \frac{1}{2}p_2^2 \\
\frac{1}{2}m_1^2 + \frac{1}{2}m_3^2 - \frac{1}{2}p_3^2 & \frac{1}{2}m_2^2 + \frac{1}{2}m_3^2 - \frac{1}{2}p_2^2 & m_3^2
\end{pmatrix}
$$

A necessary condition for a soft singularity is that for a least one value of $i$

$$
Y_{i\,i} = Y_{i\,i+1} = Y_{i\,i-1} = 0
$$

A necessary condition for a collinear singularity is that for a least one value of $i$

$$
Y_{i\,i} = Y_{i+1\,i+1} = Y_{i\,i+1} = 0
$$

If neither of these conditions is satisfied the integral is finite and can be evaluated by general techniques[1, 2, 3]

### Massless internal lines (clickable links)

1. $I_3^{\{D=4-2\epsilon\}}(0,0,p_3^2;0,0,0)$
2. $I_3^{\{D=4-2\epsilon\}}(p_1^2,p_2^2,0;0,0,0)$

### One massive internal line

The three general cases of divergent integrals are

3. $I_3^{\{D=4-2\epsilon\}}(0,p_2^2,p_3^2;0,0,m^2)$
4. $I_3^{\{D=4-2\epsilon\}}(0,p_2^2,m^2;0,0,m^2)$
5. $I_3^{\{D=4-2\epsilon\}}(0,m^2,m^2;0,0,m^2)$

### Two massive internal lines

With two massive internal lines we can only have a soft singularity. $I_4^{\{D=4-2\epsilon\}}(m_2^2,s_{12},m_3^2;0,m_2^2,m_3^2)$ is the most general divergent integral.

6. $I_3^{\{D=4-2\epsilon\}}(m_2^2,s_{12},m_3^2;0,m_2^2,m_3^2)$

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## References

[1] G. 't Hooft and M. J. G. Veltman, Nucl. Phys. B **153**, 365 (1979). Inspire

[2] G. J. van Oldenborgh, Comput. Phys. Commun. **66**, 1 (1991). Inspire

[3] T. Hahn and M. Perez-Victoria, Comput. Phys. Commun. **118**, 153 (1999) [arXiv:hep-ph/9807565]. [arXiv:hep-ph/9807565]