Divergent Triangle Integral 3: $I_3^{\{D=4-2\epsilon\}}(0,p_2^2,p_3^2;0,0,m^2)$

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The result for this integral can be extracted from ref. [1], Eq.(A.2), third equation. Analytic continuation specified by $p_i^2 \to p_i^2 + i\varepsilon$

$$I_3^{\{D=4-2\epsilon\}}(0,p_2^2,p_3^2;0,0,m^2) = \frac{1}{p_2^2-p_3^2}\left(\frac{\mu^2}{m^2}\right)^{\epsilon}\left\{\frac{1}{\epsilon}\ln\left(\frac{m^2-p_3^2}{m^2-p_2^2}\right)+\mathrm{Li}_2\left(\frac{p_2^2}{m^2}\right)-\mathrm{Li}_2\left(\frac{p_3^2}{m^2}\right)\right.$$
$$\left.+\ \ln^2\left(\frac{m^2-p_2^2}{m^2}\right)-\ln^2\left(\frac{m^2-p_3^2}{m^2}\right)\right\}+O(\epsilon)$$

For $\epsilon, \varepsilon, \ln, \mathrm{Li}_2$ see the file on notation. Rewriting this in the following form makes the $m \to 0$ limit manifest

$$I_3^{\{D=4-2\epsilon\}}(0,p_2^2,p_3^2;0,0,m^2) = \frac{1}{p_2^2-p_3^2}\left\{\frac{1}{\epsilon}\ln\left(\frac{m^2-p_3^2}{m^2-p_2^2}\right)+\frac{1}{2}\ln^2\left(\frac{-p_2^2}{\mu^2}\right)-\frac{1}{2}\ln^2\left(\frac{-p_3^2}{\mu^2}\right)\right.$$
$$\left.+\ \ln\left(\frac{(m^2-p_2^2)}{-p_2^2}\right)\ln\left(\frac{-p_2^2(m^2-p_2^2)}{\mu^2m^2}\right)-\ln\left(\frac{(m^2-p_3^2)}{-p_3^2}\right)\ln\left(\frac{-p_3^2(m^2-p_3^2)}{\mu^2m^2}\right)-\mathrm{Li}_2\left(\frac{m^2}{p_2^2}\right)+\mathrm{Li}_2\left(\frac{m^2}{p_3^2}\right)\right\}+O(\epsilon)$$

We also present this function with the singularities regulated with a small mass λ, (from ref. [1],Eq.(A.3))

$$I_3^{\{D=4\}}(0,p_2^2,p_3^2;\lambda^2,\lambda^2,m^2) = \frac{1}{p_2^2-p_3^2}\left[\mathrm{Li}_2\left(\frac{p_2^2}{m^2}\right)-\mathrm{Li}_2\left(\frac{p_3^2}{m^2}\right)+\ln^2\left(\frac{m^2-p_2^2}{\lambda m}\right)-\ln^2\left(\frac{m^2-p_3^2}{\lambda m}\right)\right]+\mathcal{O}(\lambda^2),$$

Taking the limit $m^2 \to 0$ does not introduce any singularities. We get

$$I_3^{\{D=4-2\epsilon\}}(0,p_2^2,p_3^2;0,0,0) = \frac{\mu^{2\epsilon}}{\epsilon^2}\left\{\frac{(-p_2^2-i\varepsilon)^{-\epsilon}-(-p_3^2-i\varepsilon)^{-\epsilon}}{p_2^2-p_3^2}\right\}$$
$$= \frac{1}{p_2^2-p_3^2}\left\{\frac{1}{\epsilon}\left[\ln\left(\frac{\mu^2}{-p_2^2-i\varepsilon}\right)-\ln\left(\frac{\mu^2}{-p_3^2-i\varepsilon}\right)\right]+\frac{1}{2}\left[\ln^2\left(\frac{\mu^2}{-p_2^2-i\varepsilon}\right)-\ln^2\left(\frac{\mu^2}{-p_3^2-i\varepsilon}\right)\right]\right\}+\mathcal{O}(\epsilon)$$

The expression with a mass cut-off is

$$I_3^{\{D=4\}}(p_2^2, p_3^2, 0; \lambda^2, \lambda^2, \lambda^2) = \frac{1}{p_2^2 - p_3^2}\left\{\frac{1}{2}\left[\ln^2\left(\frac{\lambda^2}{-p_2^2 - i\varepsilon}\right) - \ln^2\left(\frac{\lambda^2}{-p_3^2 - i\varepsilon}\right)\right]\right\} + \mathcal{O}(\lambda^2)$$

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References

[1] W. Beenakker, S. Dittmaier, M. Kramer, B. Plumper, M. Spira and P. M. Zerwas, Nucl. Phys. B **653**, 151 (2003) [arXiv:hep-ph/0211352]. [arXiv:hep-ph/0211352]