Divergent Triangle Integral 6: $I_3^{\{D=4-2\epsilon\}}(m_2^2, s, m_3^2; 0, m_2^2, m_3^2)$

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Before integration we have

$$I_3^{\{D=4-2\epsilon\}}(m_2^2, s, m_3^2; 0, m_2^2, m_3^2) = \frac{\Gamma(1+\epsilon)\mu^{2\epsilon}}{2\epsilon r_\Gamma} \int_0^1 d\gamma \frac{1}{\left[\gamma m_2^2 + (1-\gamma)m_3^2 - \gamma(1-\gamma)s - i\varepsilon\right]^{1+\epsilon}}$$

Introduce the notation

$$\begin{aligned}
\gamma_\pm &= \frac{1}{2}\left[\frac{m_3^2 - m_2^2 + s \pm \sqrt{(-s+m_2^2+m_3^2)^2 - 4m_2^2 m_3^2}}{s}\right] \\
\beta &= \frac{\sqrt{(-s+m_2^2+m_3^2)^2 - 4m_2^2 m_3^2}}{s} \\
x_- &= \frac{-\gamma^-}{1-\gamma^-}, \quad x_+ = \frac{\gamma^+ - 1}{\gamma^+}
\end{aligned}$$

For $s < 0$ and $m_2^2, m_3^2 > 0$ we have that $\gamma^- > \gamma^+$ and we may write the following expression,

$$\begin{aligned}
I_3^{\{D=4-2\epsilon\}}(m_2^2, s, m_3^2; 0, m_2^2, m_3^2) &= \frac{1}{2\beta s}\left(\frac{\mu^2}{-s}\right)^\epsilon \Bigg[\frac{1}{\epsilon}\ln(x_+ x_-) - \ln(-\beta)(\ln x_+ + \ln x_-) \\
&+ \frac{1}{2}\ln^2(-\gamma^+) + \frac{1}{2}\ln^2(\gamma^- - 1) - \frac{1}{2}\ln^2(\gamma^-) - \frac{1}{2}\ln^2(1-\gamma^+) \\
&- \mathrm{Li}_2\left(\frac{1-\gamma^-}{\beta}\right) - \mathrm{Li}_2\left(\frac{\gamma^+}{\beta}\right) + \mathrm{Li}_2\left(\frac{\gamma^+ - 1}{\beta}\right) + \mathrm{Li}_2\left(\frac{-\gamma^-}{\beta}\right)\Bigg] + \mathcal{O}(\epsilon)
\end{aligned}$$

This integral is also given in Eq. (6.64) of Höpker[1].

The result for the four dimensional triangle integral regulated with a small mass λ can be written, see

Beenakker and Denner[4], Eq. (C3). For $s-(m_2-m_4)^2 \neq 0$ we have

$$I_3^{\{D=4\}}(m_2^2,s,m_4^2;\lambda^2,m_2^2,m_4^2) = \frac{x_s}{m_2 m_4(1-x_s^2)}\Bigg\{\ln(x_s)\Big[-\frac{1}{2}\ln(x_s)+2\ln(1-x_s^2)+\ln(\frac{m_1 m_4}{\lambda^2})\Big]$$
$$-\frac{\pi^2}{6}+\mathrm{Li}_2(x_s^2)+\frac{1}{2}\ln^2\frac{m_2}{m_4}+\mathrm{Li}_2(1-x_s\frac{m_2}{m_4})+\mathrm{Li}_2(1-x_s\frac{m_4}{m_2})\Bigg\}+\mathcal{O}(\lambda^2)$$

For $s-(m_2-m_4)^2 = 0$ this becomes

$$I_3^{\{D=4\}}(m_2^2,s,m_4^2;\lambda^2,m_2^2,m_4^2) = \frac{1}{2m_2 m_4}\Bigg\{\ln(\frac{\lambda^2}{m_1 m_4})-2-\frac{m_4+m_2}{m_4-m_2}\ln\Big(\frac{m_2}{m_4}\Big)\Bigg\}+\mathcal{O}(\lambda^2)$$

where $x_s = -K(s+i\varepsilon,m_2,m_4)$ and K is given by

$$\begin{aligned} K(z,m,m') &= \frac{1-\sqrt{1-4mm'/[z-(m-m')^2]}}{1+\sqrt{1-4mm'/[z-(m-m')^2]}} & z \neq (m-m')^2 \\ K(z,m,m') &= -1 & z = (m-m')^2 \end{aligned}$$

For $\epsilon, \varepsilon, \ln, \mathrm{Li}_2$ see the file on notation. A special case of this integral, $I_3^{\{D=4-2\epsilon\}}(m^2,s,m^2;0,m^2,m^2)$, taken from ref.[2, 3] is given here.

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References

[1] R. Höpker, Hadroproduction and decay of squarks and gluinos, (in german), DESY Internal report DESY-T-96-02, (Relevant excerpt)

[2] W. Beenakker, S. Dittmaier, M. Kramer, B. Plumper, M. Spira and P. M. Zerwas, Nucl. Phys. B **653**, 151 (2003) [arXiv:hep-ph/0211352]. [arXiv:hep-ph/0211352]

[3] S. Dawson, R. K. Ellis and P. Nason (unpublished)

[4] W. Beenakker and A. Denner, Nucl. Phys. B **338**, 349 (1990). Inspire