Divergent Box Integral 12: $I_4^{\{D=4-2\epsilon\}}(0,m_3^2,p_3^2,p_4^2;s_{12},s_{23};0,0,m_3^2,m_4^2)$

Page contributed by R.K. Ellis

The result for this integral (see figure) is[1],

$$
\begin{aligned}
I_4^{\{D=4-2\epsilon\}}(0,m_3^2,p_3^2,p_4^2;s_{12},s_{23};0,0,m_3^2,m_4^2) &= \frac{1}{(s_{12}-m_3^2)(s_{23}-m_4^2)} \\
\times & \Bigg[\frac{1}{2\epsilon^2}-\frac{1}{\epsilon}\ln\left(\frac{(m_4^2-s_{23})(m_3^2-s_{12})}{(m_4^2-p_4^2)m_3\mu}\right)+2\ln\left(\frac{m_4^2-s_{23}}{m_3\mu}\right)\ln\left(\frac{m_3^2-s_{12}}{m_3\mu}\right) \\
- & \ln^2\left(\frac{m_4^2-p_4^2}{m_3\mu}\right)-\frac{\pi^2}{12}+\ln\left(\frac{m_4^2-p_4^2}{m_3^2-s_{12}}\right)\ln\left(\frac{m_4^2}{m_3^2}\right)-\frac{1}{2}\ln^2\left(\frac{\gamma_{34}^+}{\gamma_{34}^+-1}\right)-\frac{1}{2}\ln^2\left(\frac{\gamma_{34}^-}{\gamma_{34}^--1}\right) \\
- & 2\,\mathrm{Li}_2\Big(1-\frac{(m_4^2-p_4^2)}{(m_4^2-s_{23})}\Big)-\mathrm{Li}_2\Big(1-\frac{(m_4^2-p_4^2)}{(m_3^2-s_{12})}\frac{\gamma_{43}^+}{\gamma_{43}^+-1}\Big)-\mathrm{Li}_2\Big(1-\frac{(m_4^2-p_4^2)}{(m_3^2-s_{12})}\frac{\gamma_{43}^-}{\gamma_{43}^--1}\Big)\Bigg] \\
+ & \mathcal{O}(\epsilon)
\end{aligned}
$$

where

$$
\gamma_{ij}^{\pm}=\frac{1}{2}\Big[1-\frac{m_i^2-m_j^2}{p_3^2}\pm\sqrt{(1-\frac{m_i^2-m_j^2}{p_3^2})^2-\frac{4m_j^2}{p_3^2}}\Big]
$$

In the limit $p_3^2=0$, we get

$$
\begin{aligned}
I_4^{\{D=4-2\epsilon\}}(0,m_3^2,p_3^2,p_4^2;s_{12},s_{23};0,0,m_3^2,m_4^2) &= \frac{1}{(s_{12}-m_3^2)(s_{23}-m_4^2)} \\
\times & \Bigg[\frac{1}{2\epsilon^2}-\frac{1}{\epsilon}\ln\left(\frac{(m_4^2-s_{23})(m_3^2-s_{12})}{(m_4^2-p_4^2)m_3\mu}\right)+2\ln\left(\frac{m_4^2-s_{23}}{m_3\mu}\right)\ln\left(\frac{m_3^2-s_{12}}{m_3\mu}\right) \\
- & \ln^2\left(\frac{m_4^2-p_4^2}{m_3\mu}\right)-\frac{\pi^2}{12}+\ln\left(\frac{m_4^2-p_4^2}{m_3^2-s_{12}}\right)\ln\left(\frac{m_4^2}{m_3^2}\right)-\frac{1}{2}\ln^2\left(\frac{m_4^2}{m_3^2}\right) \\
- & 2\,\mathrm{Li}_2\Big(1-\frac{(m_4^2-p_4^2)}{(m_4^2-s_{23})}\Big)-\mathrm{Li}_2\Big(1-\frac{(m_4^2-p_4^2)}{(m_3^2-s_{12})}\Big)-\mathrm{Li}_2\Big(1-\frac{m_3^2}{m_4^2}\frac{(m_4^2-p_4^2)}{(m_3^2-s_{12})}\Big)\Bigg]+\mathcal{O}(\epsilon)
\end{aligned}
$$

See the file on notation. An associated integral is $I_4^{\{D=4-2\epsilon\}}(0,m_2^2,0,p_4^2;s_{12},s_{23};m_1^2,m_2^2,0,0)$ is given in ref. [3].

A limit of this integral, $I_4^{\{D=4-2\epsilon\}}(0,m_3^2,0,m_3^2;s_{12},s_{23};0,0,m_3^2,m_4^2)$ is given in Eq. (6.74) of ref. [2].

A limit of this integral, $I_4^{\{D=4-2\epsilon\}}(0,m_3^2,0,p_4^2;s_{12},s_{23};0,0,m_3^2,m_3^2)$ is given in Eq. (6.78) of ref. [2].

A limit of this integral, $I_4^{\{D=4-2\epsilon\}}(0,m^2,0,p_4^2;s_{12},s_{23};0,0,m^2,m^2)$ is given in ref. [4].

A limit of this integral, $I_4^{\{D=4-2\epsilon\}}(0,m^2,p_3^2,p_4^2;s_{12},s_{23},0,0,m^2,m^2)$ is given in ref. [4].

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