Divergent Box Integral 16: $I_4^{\{D=4-2\epsilon\}}(m_2^2, p_2^2, p_3^2, m_4^2; s_{12}, s_{23}; 0, m_2^2, m_3^2, m_4^2)$

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This is the general *divergent* integral with three internal masses, (see figure). Since only one internal mass is equal to zero, it can only have a soft singularity, which requires $p_1^2 = m_2^2, p_4^2 = m_4^2$ in addition to $m_1^2 = 0$. The Cayley matrix is given in this case by

$$\begin{pmatrix} 0 & 0 & \frac{1}{2}m_3^2 - \frac{1}{2}s_{12} & 0 \\ 0 & m_2^2 & \frac{1}{2}m_2^2 + \frac{1}{2}m_3^2 - \frac{1}{2}p_2^2 & \frac{1}{2}m_2^2 + \frac{1}{2}m_4^2 - \frac{1}{2}s_{23} \\ \frac{1}{2}m_3^2 - \frac{1}{2}s_{12} & \frac{1}{2}m_2^2 + \frac{1}{2}m_3^2 - \frac{1}{2}p_2^2 & m_3^2 & \frac{1}{2}m_3^2 + \frac{1}{2}m_4^2 - \frac{1}{2}p_3^2 \\ 0 & \frac{1}{2}m_2^2 + \frac{1}{2}m_4^2 - \frac{1}{2}s_{23} & \frac{1}{2}m_3^2 + \frac{1}{2}m_4^2 - \frac{1}{2}p_3^2 & m_4^2 \end{pmatrix}$$

Knowledge of the integral

$$I_4^{\{D=4-2\epsilon\}}(m_2^2, p_2^2, p_3^2, m_4^2; s_{12}, s_{23}; 0, m_2^2, m_3^2, m_4^2)$$

completely specifies the divergent boxes with three non-zero internal masses. We can calculate this IR divergent box integral from Eq. (2.9) of ref.[2], using the simple replacement rule $\ln\lambda^2 \to \frac{r_\Gamma}{\epsilon} + \ln\mu^2$. We obtain

$$\begin{aligned} I_4^{\{D=4-2\epsilon\}}(m_2^2, p_2^2, p_3^2, m_4^2; t, s; 0, m_2^2, m_3^2, m_4^2) &= \frac{x_s}{m_2 m_4 (t - m_3^2)(1 - x_s^2)} \\ \times \Bigg\{ -\frac{\ln(x_s)}{\epsilon} - 2\ln(x_s)\ln\left(\frac{m_3\mu}{m_3^2 - t}\right) &+ \ln^2(x_2) + \ln^2(x_3) - \mathrm{Li}_2(1 - x_s^2) \\ + \ \mathrm{Li}_2(1 - x_s x_2 x_3) + \mathrm{Li}_2(1 - \frac{x_s}{x_2 x_3}) &+ \mathrm{Li}_2(1 - \frac{x_s x_2}{x_3}) + \mathrm{Li}_2(1 - \frac{x_s x_3}{x_2}) \Bigg\} \end{aligned}$$

Note the reversal of the arguments s, t to conform with the notation of [2].

In the limit $x_s \to 1$ (i.e $s = (m_2 - m_4)^2$) we obtain

$$I_4^{\{D=4-2\epsilon\}}(m_2^2, p_2^2, p_3^2, m_4^2; t, s; 0, m_2^2, m_3^2, m_4^2) = \frac{1}{2m_2 m_4 (t - m_3^2)}$$
$$\times \left\{ \frac{1}{\epsilon} + 2\ln\left(\frac{m_3\mu}{m_3^2 - t}\right) - \frac{1 + x_2 x_3}{1 - x_2 x_3}\Big[\ln(x_2) + \ln(x_3)\Big] - \frac{x_3 + x_2}{x_3 - x_2}\Big[\ln(x_2) - \ln(x_3)\Big] - 2 \right\}$$

Special choices of p_2^2, p_3^2 and (non-zero) values of the masses m_2^2, m_3^2, m_4^2 will not lead to further divergences. Making the choice $m_2 = m_3 = m_4 = m$ yields the integral $I_4^{\{D=4-2\epsilon\}}(m^2, p_2^2, p_3^2, m^2; s_{12}, s_{23}; 0, m^2, m^2, m^2)$

Making the further choice $p_2^2 = 0$ yields the integral $I_4^{\{D=4-2\epsilon\}}(m^2, 0, p_3^2, m^2; s_{12}, s_{23}; 0, m^2, m^2, m^2)$

Making the further choice $p_3^2 = 0$ yields the integral $I_4^{\{D=4-2\epsilon\}}(m^2, 0, 0, m^2; s_{12}, s_{23}; 0, m^2, m^2, m^2)$

A limit of this integral, $I_4^{\{D=4-2\epsilon\}}(m_1^2, 0, 0, m_1^2; s_{12}, s_{23}; 0, m_1^2, m_2^2, m_1^2)$ is given in Eq. (6.73) of ref. [1].

A limit of this integral, $I_4^{\{D=4-2\epsilon\}}(m_1^2, 0, 0, m_2^2; s_{12}, s_{23}; 0, m_1^2, m_1^2, m_2^2)$ is given in Eq. (6.76) of ref. [1].

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References

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