We work in the Bjorken-Drell metric so that \( l^2 = l_0^2 - l_1^2 - l_2^2 - l_3^2 \). The definition of the integrals are as follows

\[
I_{4}^{(D)}(p_1^2, p_2^2, p_3^2, p_4^2; (p_1 + p_2)^2, (p_2 + p_3)^2; m_1^2, m_2^2, m_3^2, m_4^2) = \frac{\mu^{4-D}}{i\pi^{2} r_{\Gamma}} \int d^{D}l \frac{1}{(l^2 - m_1^2 + i\varepsilon)((l + q_1)^2 - m_2^2 + i\varepsilon)((l + q_2)^2 - m_3^2 + i\varepsilon)((l + q_3)^2 - m_4^2 + i\varepsilon)}
\]

\[
I_{3}^{(D)}(p_1^2, p_2^2, p_3^2; m_1^2, m_2^2, m_3^2) = \frac{\mu^{4-D}}{i\pi^{2} r_{\Gamma}} \int d^{D}l \frac{1}{(l^2 - m_1^2 + i\varepsilon)((l + q_1)^2 - m_2^2 + i\varepsilon)((l + q_2)^2 - m_3^2 + i\varepsilon)}
\]

\[
I_{2}^{(D)}(p_1^2; m_1^2, m_2^2) = \frac{\mu^{4-D}}{i\pi^{2} r_{\Gamma}} \int d^{D}l \frac{1}{(l^2 - m_1^2 + i\varepsilon)((l + q_1)^2 - m_2^2 + i\varepsilon)}
\]

\[
I_{1}^{(D)}(m_1^2) = \frac{\mu^{4-D}}{i\pi^{2} r_{\Gamma}} \int d^{D}l \frac{1}{(l^2 - m_1^2 + i\varepsilon)}
\]

where \( q_1 = p_1, q_2 = p_1 + p_2, q_3 = p_1 + p_2 + p_3 \) and \( q_0 = q_4 = 0 \).

Near four dimensions we use \( D = 4 - 2\epsilon \). The symbol \( \epsilon \) is thus equal to \( \frac{4-D}{2} \). (For clarity the small imaginary part which fixes the analytic continuations is specified by \( +i\varepsilon \)). The overall constant which occurs in \( D \)-dimensional integrals is,

\[
r_{\Gamma} = \frac{\Gamma^2(1 - \epsilon)\Gamma(1 + \epsilon)}{\Gamma(1 - 2\epsilon)} = \frac{1}{\Gamma(1 - \epsilon)} + O(\epsilon^3).
\]

Since

\[
\Gamma(1 - \epsilon) = 1 + \epsilon \gamma + \epsilon^2 \left[ \frac{\gamma^2}{2} + \frac{\pi^2}{12} \right] + O(\epsilon^3)
\]

\[
r_{\Gamma} = 1 - \epsilon \gamma + \epsilon^2 \left[ \frac{\gamma^2}{2} - \frac{\pi^2}{12} \right] + O(\epsilon^3).
\]

Various useful formula for dimensional regularization are given here.
The final results are given in terms of logarithms and dilogarithms. The logarithm is defined to have a cut along the negative real axis. The rule for the logarithm of a product is

\[ \ln(ab) = \ln a + \ln b + \eta(a, b) \]

\[ \eta(a, b) = 2\pi i\left[\theta(-\text{Im}(a))\theta(-\text{Im}(b))\theta(\text{Im}(ab)) - \theta(\text{Im}(a))\theta(\text{Im}(b))\theta(-\text{Im}(ab))\right] \]

So that

\[ \ln(ab) = \ln a + \ln b, \text{ if } \text{Im}(a) \text{ and } \text{Im}(b) \text{ have different signs.} \]

\[ \ln\left(\frac{a}{b}\right) = \ln a - \ln b, \text{ if } \text{Im}(a) \text{ and } \text{Im}(b) \text{ have the same sign.} \]

The dilogarithm is defined as

\[ \text{Li}_2(x) = -\int_0^x \frac{dx}{x} \ln(1 - x) = \frac{x}{1^2} + \frac{x^2}{2^2} + \frac{x^3}{3^2} + \ldots \text{ when } |x| \leq 1 \]

A number of the most commonly useful dilogarithm identities are given [here](#).

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